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Stationkeeping Method for Libration Point Trajectories

K. C. Howell* and H. J. Pernicka†
Purdue University, West Lafayette, Indiana 47907

Three-dimensional orbits in the vicinity of the interior libration point (L_1) of the Sun-Earth/Moon barycenter system are currently being considered for use with a number of missions planned for the 1990s. Since such libration point trajectories are, in general, unstable, spacecraft moving on these paths must use some form of trajectory control to remain close to their nominal orbit. The primary goal of this effort is the development of a stationkeeping strategy applicable to such trajectories. A method is presented that uses maneuvers executed (impulsively) at discrete time intervals. The analysis includes some investigation of a number of the problem parameters that affect the overall maneuver costs. Simulations are designed to provide representative stationkeeping costs for a spacecraft moving in a libration point trajectory, and preliminary results are summarized.

Introduction

TRAJECTORY planning for a number of scientific missions scheduled for launch in the 1990s includes the possible use of three-dimensional halo or Lissajous orbits in the vicinity of the interior L_1 libration point of the Sun-Earth/Moon barycenter system.[1] This effort is directed toward the development of a stationkeeping strategy that can be used to maintain spacecraft near such nominal libration point trajectories. A significant number of analyses have been completed that involve stationkeeping methods for Earth orbiting satellites; maintaining a spacecraft on a libration point orbit, however, has received limited attention. In the late 1960s, Farquhar[2] developed a number of possible stationkeeping strategies for libration point orbits. Later, in 1974, a stationkeeping method for spacecraft moving on halo orbits in the vicinity of the Earth-Moon translunar libration point L_2 was published by Breakwell et al.[3] In contrast, specific mission requirements influenced the design of the stationkeeping strategy for the first libration point mission. When the International Sun-Earth Explorer-3 (ISEE-3) satellite was injected into a halo orbit associated with the interior L_1 libration point of the Sun-Earth system in 1978, a series of maneuvers executed at approximately three-month intervals was used for stationkeeping.[4] More recently, a series of papers have presented results from studies that use Floquet and invariant manifold theories to develop a "loose" stationkeeping strategy for halo-type orbits.[5–8] Similar to ISEE-3, the method also uses discrete maneuvers, applied at varying time intervals, that control the trajectory near the nominal path. A significant goal of this study is the development of a potentially "tight" control strategy for stationkeeping that can be applied to both halo and Lissajous trajectories (as well as other possible types of libration point paths). In this approach, the allowable deviation of the actual trajectory relative to the nominal path can be varied over a wide range depending on mission requirements. Of course, low costs are desirable as well.

Halo and Lissajous Trajectories

In previous studies, halo and Lissajous trajectories have been numerically determined in the circular and elliptic restricted three-body problems.[7,9–13] Halo orbits have been calculated numerically from exact nonlinear equations of motion using differential corrections schemes.[9–11] The numerical computation of Lissajous trajectories is complicated, in part, by their nonperiodicity. However, Howell and Pernicka[12,13] have developed a "patching" technique in connection with a differential corrections scheme that results in the numerical determination of Lissajous trajectories of arbitrary size and length.

Determination of more accurate trajectories, however, requires the capability to numerically compute halo and Lissajous orbits for nonperiodic primary motion. Some recent studies[7,13] have developed (different) techniques for the numerical determination of halo-type trajectories with models that employ nonperiodic primary motion. The resulting paths are not precisely periodic and thus are not true halo orbits as originally defined. However, their shape and size will typically satisfy mission design requirements when halo orbits are required. One example of a (numerical) "near-halo" orbit is shown in Fig. 1, using the technique described in Ref. 13. This orbit is located in the vicinity of the L_1 libration point associated with the Sun-Earth/Moon barycenter system and was computed with a model that includes the gravitational forces from the Sun, Earth, and Moon (and is subsequently identified as the "SEM" model). The trajectory is computed using a numerical integration procedure that includes determination of the locations of the Sun, Earth, and Moon from polynominal representations of ephemeris data. Three planar projections of the orbit are shown, with the origin in each plot corresponding to the L_1 point. The three axes associated with the figure are defined consistent with the rotating frame typically used with the restricted three-body problem. Thus, the x axis is directed from the larger primary (Sun) to the smaller (Earth/Moon barycenter), the y axis is defined in the plane of the orbit of the primaries 90 deg from the x axis, and the z axis completes the right-handed frame. [It should be noted that the L_1 point cannot actually be found for the system as modeled here, since the motion of the primaries is not periodic. However, it is desirable to display trajectories in terms of coordinates corresponding to the usual rotating frame with origin at L_1. This is accomplished by using the (known) location of L_1 in the elliptic restricted problem to compute a corresponding approximate location for the libration point in the system defined by the SEM model.] The orbit shown has an approximate out-of-plane amplitude $A_z = 120{,}000$ km, which requires the corresponding approximate in-plane amplitudes $A_y = 658{,}000$ km and $A_x = 203{,}000$ km. Each revolution of the orbit has a duration of approximately 6 months, and the trajectory is shown for a total of 13 revolutions (6.33 yr). Note that the spacecraft moves on its path about L_1 in a counterclockwise direction as viewed in the y-z plane.

The numerical determination of Lissajous trajectories associated with a model incorporating nonperiodic primary mo-

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*Associate Professor, School of Aeronautics and Astronautics. Senior Member AIAA.

†Graduate Student, School of Aeronautics and Astronautics; current address: Aerospace Engineering, San Jose State University, One Washington Square, San Jose, CA 95192-0188. Member AIAA.

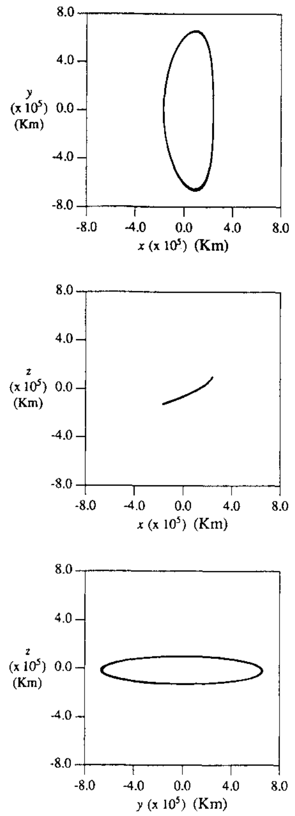

Fig. 1 Example of numeric near-halo trajectory.

tion has recently been considered.[13] An example of such a Lissajous trajectory is shown in Fig. 2 and was computed using the SEM model for primary locations. Similar to the orbit shown in Fig. 1, this trajectory is also located in the vicinity of the L_1 libration point associated with the Sun-Earth/Moon barycenter system. Three planar projections of the orbit are shown, with the origin in each plot corresponding to the L_1 point. The duration of the path is 6.28 yr, with approximate amplitudes $A_z = 500{,}000$ km and $A_y = 400{,}000$ km. Unlike a halo orbit, A_z and A_y may be selected independently. The spacecraft moves on the Lissajous path about L_1 in a counterclockwise direction as viewed in the y–z plane.

Stationkeeping Problem

Definition

One of the main goals of the trajectory design process is determination of the nominal path. Once the nominal orbit is determined, it is desired to maintain the actual spacecraft trajectory within some torus centered about the "reference" path. It is not required that the spacecraft remain precisely on the nominal path. Unmodeled perturbations as well as any orbit injection error will result in drift of the spacecraft from the nominal path, and the unstable nature of libration point orbits will further amplify the deviation. Thus, a stationkeeping strategy is required that will compute maneuvers to be performed at specified times to drive the spacecraft trajectory back to the acceptable region within the torus. Assuming discrete, impulsive corrections, the problem consists of the determination of any required corrective maneuvers (including the magnitude, direction, and timing of each ΔV). Of course, reduction of maneuver magnitudes is part of the problem as well.

Nominal Orbit Selection

Maneuver costs associated with stationkeeping may be significantly greater for spacecraft attempting to follow nominal paths computed from inaccurate models. Therefore, selection of the nominal orbit can be critically important. One obvious choice is to compute the nominal orbit with a model that most closely represents the actual forces acting on the spacecraft. For purposes of this study, then, the SEM model for motion of the primaries was used to compute the nominal path (i.e., the SEM model was considered "the best" model available). In particular, the orbit selected for use as the nominal path in most of the stationkeeping simulations, shown in Fig. 1, is close in size to the halo trajectory used in the ISEE-3 mission (and is, in fact, similar to one considered for use with another mission scheduled for the mid-1990s). Choice of a nominal path that is very close to a familiar halo orbit offers the opportunity to compare the stationkeeping results from this study with other stationkeeping analyses published in association with work on halo orbits.[5-8] (It should be noted, however, that the stationkeeping strategy developed here does not differentiate between halo and Lissajous trajectories and can be applied to either.)

Representation of the Nominal Orbit

For the stationkeeping strategy developed here (and in other approaches as well), the nominal orbit and its state transition matrix must be available (i.e., obtainable at any time along the orbit) in a convenient manner. Inaccurate representations may contribute to higher maneuver costs. Thus, methods were sought that would reproduce the nominal orbit information accurately at an "affordable" computational cost.

When the nominal libration point orbit is (numerically) computed, output data is generated in the form of a tabular listing of times and corresponding position and velocity states. A number of methods were considered to represent this data in a form such that, given a value of time, the nominal position and velocity of the spacecraft could be easily obtained. During the early phases of this work, Fourier series were used to write position and velocity as a function of time (an obvious choice, since plots of x, y, or z vs time appear sinusoidal in shape for either a halo or Lissajous trajectory). The accuracy of these curve fits depends, of course, on the number of terms retained in the series. The precise effects of the number of coefficients

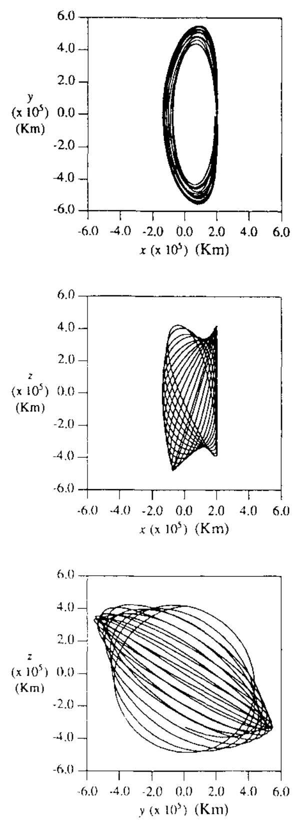

Fig. 2 Example of numeric Lissajous solution.

on accuracy was unclear, and a significant amount of computational time and storage was required to produce and retain the coefficients. Because of these difficulties, more convenient methods were desirable. Out of a number of different approaches that were investigated, the most successful method employs a technique that represents the nominal orbit by an Akima cubic spline interpolation. Storing the nominal path in this form provides a very accurate and convenient representation (errors typically less than 0.1 km in position and 1.0 mm/s in velocity for data intervals up to 1.1 days in length). However, it was determined that stationkeeping costs did begin to rise significantly (as a result of loss in accuracy of the nominal orbit representation) if the data points were spaced in intervals of greater than 1.1 days.

Unfortunately, the size of the data files precludes use of this method to represent the 36 elements of the state transition matrix, and another approach was sought. A number of methods were investigated. Ultimately, instead of attempting to represent each element, the simulation software was written to reproduce the state transition matrix associated with the nominal orbit, as needed, by numerically integrating the variational equations assuming primary motion consistent with the SEM model. It is noted that this approach depends heavily on an accurate representation of the nominal orbit.

Algorithm

Additional Assumptions

To test a particular stationkeeping strategy, an algorithm must first be developed that simulates any conditions under which the spacecraft might operate that would affect the trajectory. Possibilities include the system of gravitational forces acting on the spacecraft, the spacecraft injection maneuver into the libration point orbit, the spacecraft tracking process, mission constraints, the execution of propulsive maneuvers, as well as numerous other (modeled and/or unmodeled) forces, both continuous and single "impulsive" events. For the simulation algorithm used in this study, no attempt has been made to include all of the conditions that could affect the spacecraft's actual trajectory. The simulation algorithm has been developed solely to provide a tool that could be used to evaluate possible stationkeeping strategies. The focus has thus been restricted to consideration of the impact of the injection, tracking, and maneuver processes on a nominal orbit determined in the presence of gravity from the Sun, Earth, and Moon.

To precisely simulate the spacecraft trajectory, the SEM model for primary motion in the restricted three-body problem is used to propagate some set of initial conditions specified for the spacecraft. The initial position and velocity of the spacecraft immediately after a maneuver to inject the spacecraft into the libration point orbit is used to define the initial conditions. In addition, this set of initial conditions can be altered to simulate an orbit injection error. The subsequent simulated motion is then defined to be the "actual" spacecraft trajectory. The algorithm that includes the SEM model is of a general nature, and additional perturbations (attracting masses and solar radiation pressure, for example) can be added as needed. To represent tracking data, the spacecraft position and velocity is recorded based on an adjustable time interval that reflects typical lengths of time between Earth-based tracking readings (denoted as "tracking intervals"). As the simulation proceeds, corrective maneuvers (modeled as impulsive) can be implemented by simply changing the spacecraft velocity states at the specified time. The simulation continues to the predetermined stop time. (However, if the trajectory deviates unacceptably far from the nominal path, the simulation is terminated immediately. This study defines 50,000 km as the termination distance.)

Orbit Injection, Tracking, and Maneuver Errors

A transfer trajectory originating near the Earth or some other location can be used to place a spacecraft into a libration point orbit. When the spacecraft arrives at its target location near the libration point, a maneuver is performed to "insert" into the nominal orbit with the required velocity. Because of various error sources, the spacecraft will be injected with some inaccuracy in both its position and velocity states. The simulations mimic this injection error by using a random number generator corresponding to a Gaussian probability distribution to compute a value to represent the error in each component of the state $\{x, y, z, \dot{x}, \dot{y}, \dot{z}\}$. The mean is assumed to be zero, and the specified value of the standard deviation σ "shapes" the distribution curve. (A value for σ can be obtained from previous experience with actual hardware.) When the simulation begins, the six values corresponding to the initial position and velocity on the nominal orbit are randomly changed. (Six individual computations are made so that each component of position and velocity can be changed by a different amount.) Thus, the simulated injection error is random in direction and magnitude for each component of both position and velocity.

As the spacecraft moves on its orbit about the libration point, Earth-based stations track the vehicle and compute its position and velocity. Because of errors in this orbit determination process, the spacecraft position and velocity are never exactly known. The selected stationkeeping strategy must maintain the satellite near its nominal path in spite of these errors, at a reasonable cost. To simulate the tracking errors, the same method used to compute the orbit injection error is applied. The integration scheme (using the differential equations derived assuming primary motion consistent with the SEM model) produces the "actual" spacecraft position and velocity at constant tracking intervals (typically 2 days as assumed here). When the position or velocity that would normally be obtained from tracking data is required during the simulation, the actual location and velocity produced by the integration procedure is altered by randomly applying an error to each of the six states. As before, the mean is assumed to be zero and σ determines the approximate magnitude of the tracking error.

At various times along the trajectory, the stationkeeping strategy will determine that a maneuver is required. The magnitude and direction is then computed, and the maneuver is "executed." Of course, the actual maneuver that is implemented will not change the velocity of the spacecraft by the exact amount specified. To model this inaccuracy, the approach used for simulating the injection and tracking errors is again employed, in which each ΔV component is altered randomly by use of a Gaussian probability distribution. Whenever the simulation computes a maneuver to be executed, the three components of the computed ΔV are randomly changed before implementation. As usual, the mean is assumed zero and σ determines the approximate magnitude of the maneuver error.

Stationkeeping Strategy

After considering various algorithms discussed in the literature for possible application to the stationkeeping problem, an approach presented by Dwivedi[14] has been modified and successfully used to control the spacecraft trajectory near the nominal path. Although originally applied to the problem of interplanetary targeting, the theory appears to be adaptable to the libration point orbit stationkeeping problem.

As applied to the stationkeeping problem, the approach begins by defining a cost function. For convenience, the nominal 6×6 state transition matrix $\Phi(t_2, t_1)$ is first partitioned as

$$\Phi(t_2, t_1) = \begin{bmatrix} A_{21} & B_{21} \\ C_{21} & D_{21} \end{bmatrix} \tag{1}$$

where A_{21}, B_{21}, C_{21}, and D_{21} are all 3×3 submatrices. Let $\boldsymbol{m}_{t_i}$ be a 3×1 vector representing the deviation of the actual trajectory from the nominal path at time t_i due to a velocity perturbation $\boldsymbol{e}(t_o)$ at time t_o, a position perturbation $\boldsymbol{p}(t_o)$ at time t_o, as well as a corrective maneuver $\Delta V_c(t)$ performed at some time $t \geq t_o$. The index i represents a particular target point, and $e(t_o)$, $p(t_o)$, and $\Delta V_c(t)$ are 3×1 vectors. Then, t_i is some future time at which the predicted actual path will be compared to the nominal. Target times are actually identified by specification of the input variable Δt_i, i.e., their values relative to t_o, such that $t_i = t_o + \Delta t_i$. Initially consider the case of two target points. With these definitions, the linear approximation for the position deviation at target i may be expressed as

$$m_{t_i} \cong B_{t_i t_o} e(t_o) + B_{t_i t} \Delta V_c(t) + A_{t_i t_o} p(t_o) \tag{2}$$

Developed for application to interplanetary trajectories, Dwivedi's approach includes the assumption that the position perturbations will have a negligible effect when compared to the velocity perturbations. Consequently, the position perturbation $p(t_o)$ is assumed negligible and is not included. However, for application to a libration point trajectory, the effects of position deviations can be significant. An examination of the state transition matrix shows that the position and velocity sensitivities can be of the same order of magnitude. Thus, an effective stationkeeping algorithm requires the inclusion of the effect of the position error $p(t_o)$ in Eq. (2). The cost function is then defined such that

$$J[p, e, \Delta V_c(t)] = \Delta V_c(t)^T Q(t) \Delta V_c(t) + m_{t_1}^T R(t) m_{t_1} + m_{t_2}^T S(t) m_{t_2} \tag{3}$$

where superscript T denotes transpose. The weighting matrix Q is symmetric positive definite, and R and S are positive semidefinite.

Computing an optimal maneuver then consists of minimizing the cost function shown in Eq. (3). The result produces the desired optimal maneuver as

$$\Delta V_c(t) = -[Q(t) + B_{1t}^T R(t) B_{1t} + B_{2t}^T S(t) B_{2t}]^{-1} \times \{[B_{1t}^T R(t) B_{1t_o} + B_{2t}^T S(t) B_{2t_o}] e(t_o) + [B_{1t}^T R(t) A_{1t_o} + B_{2t}^T S(t) A_{2t_o}] p(t_o)\} \tag{4}$$

Equation (4) has been employed successfully in this work. However, for this preliminary investigation of the approach a number of additional assumptions were made that should be carefully noted. For this study, all maneuvers have been computed and assumed to occur at $t = t_o$. However, future studies might investigate the possibility of some optimal maneuver time t_c. Equation (4) is also based on the use of two future target times which is an arbitrary choice. The theory can be extended to any number, but it is not known if such an extension is helpful. In addition, the weighting matrices Q, R, and S have been assumed as constant for this work, i.e., the elements of each matrix have been held constant during a simulation. Future work will examine the possibility of using time-varying matrices. (Some experimentation has confirmed that tailoring the elements for different simulations may be useful.)

Preliminary Results

The initial evaluation of the stationkeeping algorithm based on Eq. (4) for computation of each corrective measure has been carried out in a four-step process. First, the total parameter set associated with the stationkeeping problem was identified. Second, a set of "initial" input values for all parameters was found (generally by trial-and-error) that would result in successful control of the spacecraft near its nominal path. Third, some attempts were made to reduce the maneuver cost by adjusting the "initial set" of input parameter values. The set of parameter input values associated with the lowest cost was then defined as the "baseline" set for the remainder of the study. Finally, a subset of the total set of parameters was selected for further study. In particular, numerous simulations were performed to investigate the parameters in the subset and their (individual) effects on total cost.

Parameter Identification

The method developed to approach the stationkeeping problem involves many different parameters that influence the perturbations affecting the spacecraft and the resulting maneuver costs. Some include the following:

1) The number and spacing of future target times.
2) The particular values for elements of the weighting matrices.
3) The minimum time required between stationkeeping maneuvers.
4) The magnitude and direction of the libration point orbit injection error.
5) The magnitude and direction of tracking errors.
6) The magnitude and direction of maneuver errors.
7) Errors inherent in the model used to generate the "actual" trajectory.
8) The determination of some "optimal" time for a stationkeeping maneuver.

Although the effects of all of these parameters were not analyzed, a number were considered in some detail to investigate the change in spacecraft drift and maneuver costs due to varying inputs.

Numerous stationkeeping simulations have been performed for the nominal orbit shown in Fig. 1. The stationkeeping algorithm uses Eq. (4) to compute the corrective maneuvers. Three additional constraints are also specified that impact the timing of each maneuver. First, an input parameter $t_{\min}$ is defined as a minimum length of time that must elapse from the last maneuver before another can be performed. This constraint is usually due to orbit determination requirements and/or scientific payload specifications. Second, another constraint requires that the drift of the spacecraft from the nominal path exceed a specified amount $d_{\min}$ before a maneuver is executed. A maneuver is performed only if $d > d_{\min}$. A final criterion prevents a maneuver from being executed if the current deviation "d" from the nominal path is decreasing (i.e., the spacecraft must be drifting farther from the nominal orbit for a maneuver to be done). Thus, whenever all three criteria are simultaneously met, a corrective maneuver $\Delta V_c(t)$ is computed and implemented.

Baseline Cases

The second step in evaluation of the stationkeeping strategy was determination of some set of input values to the algorithm that would result in "reasonable" values for spacecraft drift and total maneuver cost. In other words, it was necessary to obtain "baseline" values for all of the parameters. Then the effects of varying certain parameters relative to their baseline values could be investigated. Prior to determining a final set of baseline values, some "initial" set was required that would succeed in keeping the spacecraft near the nominal path, regardless of the maneuver cost. (The unstable nature of these orbits implies that an arbitrary set of input values may result in a trajectory that diverges from the nominal path in a short period of time.) Certain parameter values were predetermined by mission specifications. This "initial" set of parameters and their corresponding values include the following:

1) Duration of each simulation = 2 yr.
2) Halo orbit injection on July 1, 1995.
3) Tracking information received at 2-day intervals.
4) Weighting matrix $Q_o = \mathrm{diag}(1 \times 10^{13}, 1 \times 10^{13}, 1 \times 10^{13})$.
5) Weighting matrix $R_o = \mathrm{diag}(1, 1, 1)$.
6) Weighting matrix $S_o = \mathrm{diag}(1, 1, 1)$.
7) Standard deviation for orbit injection error and tracking errors: $\sigma = 1.5, 2.5, 15.0$ km for x, y, z components, respectively; $\sigma = 1.0, 1.0, 3.0$ mm/s for $\dot{x}$, $\dot{y}$, $\dot{z}$ components, respectively.

8) Standard deviation for maneuver errors: $\sigma = 2.5\%$ of the planned maneuver magnitude, applied to $\dot{x}$, $\dot{y}$, $\dot{z}$ components.

Note that the units for elements of the weighting matrices Q, R, and S were chosen to correspond with ΔV values in units of meters per second and position deviations expressed in terms of kilometers. The σ values were selected to roughly correspond to values used in other published stationkeeping analyses.[5-8] Also, a near-halo orbit of 2-yr duration was initially used that is slightly different from the trajectory shown in Fig. 1. A nominal path of shorter duration greatly reduced the required computing time. Once the baseline input values were determined, the remaining simulations were performed using the nominal path shown in Fig. 1.

The simulations were divided into three groups based on t_{min}, the minimum time required between maneuvers. Three values of t_{min} were selected (not totally arbitrarily) and the groups defined as group A ($t_{min} = 30$ days), group B ($t_{min} = 60$ days), and group C ($t_{min} = 80$ days). For each of these groups, a number of trial-and-error runs were initially performed in an effort to determine some best pair of times Δt_1 and Δt_2, relative to t_o. The goal was to locate the two target points such that the ΔV costs are as low as possible given initial values of d_{min}, Q, R, and S. These resulting target locations are not necessarily optimal, but they give a reasonable starting point for examining the effects of varying other parameters on the stationkeeping cost. Note that when computing the injection error, the same "seed" to the random number generator was used so that the injection error is identical for each simulation, making comparison of the results more meaningful. However, direct comparisons cannot necessarily be made since some randomness still remains in the tracking and maneuver error computations. This issue is addressed in detail shortly.

Once the target times Δt_1 and Δt_2 were identified, the value of d_{min} (the minimum departure from the nominal path required before corrective action can be taken) was varied to further reduce the total cost. Lastly, given the locations for target points and a "best" d_{min}, the diagonal elements of the weighting matrices Q, R, and S were varied to again reduce the maneuver costs if possible. The final combination of times Δt_1 and Δt_2; distance d_{min}; and values for the diagonal elements of Q, R, S is, of course, not necessarily optimal but it did provide an acceptable starting point. Table 1 summarizes the results of simulations performed to determine baseline values of Δt_1, Δt_2, d_{min}, Q, R, and S for groups A, B, and C as just outlined. The table is organized by columns for the three values of t_{min} and lists the final values that were obtained from the sequence of trial-and-error attempts that resulted in the lowest maneuver cost for each case. The first three lines of the table provide the baseline values obtained for Δt_1, Δt_2, and d_{min}, assuming weighting matrices given as Q_o, R_o, S_o. The weighting matrices were then altered over a number of trials to determine a combination of weights that would most reduce costs. The remainder of the table shows the adopted baseline values for Q, R, and S, along with the reduced stationkeeping costs. Although the same Q_o, R_o, and S_o were initially used for each group A, B, and C, the resulting baseline matrices Q, R, and S were different between groups. Thus, different choices for the weights can significantly affect the cost.

Effects of Varying Input Parameters

To investigate effects of varying other parameters on the stationkeeping costs, the baseline values shown in Table 1 for Δt_1, Δt_2, d_{min}, Q, R, and S were fixed for the remaining trials. The results for the subsequent simulations are summarized in Table 2, and presented in three columns corresponding to groups A, B, C ($t_{min} = 30$, 60, and 80 days). Each simulation was performed for 6 yr using the near-halo orbit shown in Fig. 1. (It is assumed that the baseline values obtained for the 2-yr halo orbit still provide a set of inputs for the trials with the 6-yr path that result in "low" costs. Future efforts could involve development of a more sophisticated approach for selection of baseline inputs.) Note that the ΔV costs listed in

Table 1 Determination of input parameters and associated costs

	Nominal halo orbit: 2.44-yr trajectory $A_z = 110{,}000$ km 2-yr simulations		
	Group A $t_{min} = 30$ days	Group B $t_{min} = 60$ days	Group C $t_{min} = 80$ days
Δt_1, days	40	65	110
Δt_2, days	65	95	140
d_{min}, km	0.0	0.0	100
Q	Diag(5.0×10^{12}, 3.0×10^{13}, 1.0×10^{13})	Diag(1.0×10^{12}, 1.0×10^{13}, 8.0×10^{12})	Diag(1.0×10^{13}, 1.3×10^{13}, 1.0×10^{13})
R	Diag(1.0, 0.0, 1.0)	Diag(0.0, 1.0, 1.1)	Diag(5.0, 1.0, 100)
S	Diag(1.0, 1.0, 1.0)	Diag(1.7, 1.0, 1.0)	Diag(1.0, 0.85, 0.6)
Baseline cost, m/s	0.242 (Case A_{BL})	0.449 (Case B_{BL})	0.484 (Case C_{BL})

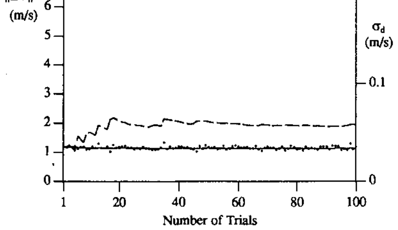

Fig. 3 Average maneuver costs for case A1.

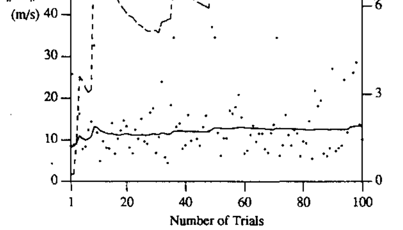

Fig. 4 Average maneuver costs for case C1.

Table 2 Summary of stationkeeping cost as a function of input parameters

Nominal halo orbit: 6.33-yr trajectory
$A_y = 658{,}000$ km
$A_z = 120{,}000$ km
Maneuver costs: average of 100 trials

Input parameters	Total ΔV cost, m/s Group A $t_{min} = 30$ days	Group B $t_{min} = 60$ days	Group C $t_{min} = 80$ days
Baseline cost, m/s	1.129 (Case A1)	2.450 (Case B1)	13.386 (Case C1)
ΔV_{min}:			
0.01 m/s	1.050 (A2)	2.433 (B2)	13.386 (C2)
0.05 m/s	1.931 (A3)	2.412 (B3)	12.387 (C3)
0.10 m/s	3.125 (A4)	3.089 (B4)	10.490 (C4)
0.50 m/s	12.738 (A5)	11.445 (B5)	16.640 (C5)
Tracking interval:			
1 day	1.083 (A6)	2.391 (B6)	18.973 (C6)
2 days	1.129 (A1)	2.450 (B1)	13.386 (C1)
4 days	1.288 (A7)	2.463 (B7)	12.008 (C7)
8 days	1.578 (A8)	2.863 (B8)	11.274 (C8)
Injection errors:			
M.F. = 20	1.433 (A9)	3.600 (B9)	33.123 (C9)
M.F. = 40	1.853 (A10)	4.979 (B10)	51.418 (C10) [4]
M.F. = 60	2.301 (A11)	6.493 (B11)	78.633 (C11) [5]
M.F. = 80	2.770 (A12)	7.910 (B12)	94.242 (C12) [17]
M.F. = 100	3.263 (A13)	9.390 (B13)	102.895 (C13) [20]
Tracking errors:			
M.F. = 20	7.502 (A14)	13.353 (B14)	98.862 (C14) [21]
M.F. = 40	15.028 (A15)	26.414 (B15)	133.982 (C15) [73]
M.F. = 60	22.406 (A16)	39.346 (B16)	
M.F. = 80	29.939 (A17)	52.639 (B17)	
M.F. = 100	37.251 (A18)	65.562 (B18)	
Maneuver errors:	Baseline error multiplied by:	Baseline error multiplied by:	Baseline error multiplied by:
	2→ 1.153 (A19)	2→ 3.117 (B19)	2→ 56.259 (C19)
	4→ 1.235 (A20)	4→ 23.425 (B20) [5]	4→ 73.601 (C20) [99]
	8→ 1.797 (A21)	6→ 51.384 (B21) [81]	
	16→ 113.164 (A22) [76]		

Tables 1 and 2 do not actually represent the same type of information. Each ΔV listed in Table 1 corresponds to the result from a single simulation. Since the cost of a single simulation may be affected significantly by the randomness generated in the modeled errors (by either increasing or decreasing the cost), direct comparisons between cases cannot always be made. In Table 2, however, each tabulated number represents the average total ΔV cost (in meters per second) obtained from 100 simulations (with some exceptions noted shortly), each using identical parameter inputs. The trials are differentiated by use of a different seed to initialize the random-number generator, thus producing a different sequence of random numbers used to compute the injection, tracking, and maneuver errors. In this manner, the value obtained for the total maneuver cost is more representative in general. Also, comparisons between cases now have more significance. However, the number of simulations required to produce a representative number is not always obvious, and depends, of course, on the values of the input parameters.

A number of simulations in each group A, B, and C were initially performed for baseline inputs and their data analyzed. In Table 2, the section labeled "Baseline Cost" lists the average costs obtained over 6 yr for all three groups using the baseline parameter inputs. Each total cost figure was obtained by computing the average from 100 simulations, changing only the input "seed" between trials. As examples, the results from simulations using baseline inputs that correspond to two cases (cases A1 and C1 in the table) are displayed in Figs. 3 and 4. The abscissa variable is the number of simulations, the left-hand ordinate measures the total maneuver cost (meters per second), and the right-hand ordinate reflects the standard deviation σ_d of the data (meters per second). In the figures, a single dot represents the total maneuver cost for a single simulation. The solid line then represents the cumulative average of the total maneuver cost. Note that the average maneuver cost corresponding to 100 trials in each figure is the value in the table for each case A1 and C1. The dashed line shows the cumulative standard deviation associated with the total maneuver costs. The average in case A1, shown in Fig. 3, "settles" rapidly: after 15 trials, the average changes by less than 2% of the current average. For this case, then, 100 simulations is probably sufficient to obtain a representative cost. (A simple estimate of the number of runs required indicates that 100 simulations is more than sufficient for groups A and B.) Figure 4 shows that after about 50 trials, the average for case C1 changes by less than 1 m/s (less than 10% of the current average). The cumulative standard deviation, however, has not reached a constant value after 100 trials. In this case, 100 simulations may not be sufficient. However, it should provide an indication of the order of magnitude of the resulting cost. The maneuver costs cited in Table 2 were all obtained using 100 simulations to compute the averages.

Comparing the baseline costs in cases A1, B1, and C1 reveal that the stationkeeping ΔV requirements approximately double from case A1 to B1, and increase by a factor of more than five from case B1 to C1. Thus, increasing t_{min} beyond a certain limit apparently has a significant effect on cost. The values displayed in the remaining sections of Table 2 demonstrate the effects of a number of other parameters, varied one at a time, and can be compared to the baseline values.

The first parameter considered was ΔV_{min}, i.e., an estimate of the smallest possible ΔV that can be implemented by the

Table 3 Stationkeeping time history: case A4

Case A4: $t_{min} = 30$ days, $\Delta V_{min} = 0.1$ m/s							
Time, days	Δt, days	ΔV_x, m/s	ΔV_y, m/s	ΔV_z, m/s	$\|\Delta V\|$, m/s	Maneuver error, %	Deviation, km
132.1	132.1	0.099	0.020	−0.001	0.101	1.2755	119.78
206.1	74.1	0.107	0.004	0.002	0.107	0.6648	117.82
274.2	68.0	0.097	0.022	−0.009	0.100	3.1340	135.39
360.2	86.1	0.107	0.004	0.006	0.108	2.9862	143.46
424.3	64.0	−0.101	−0.010	0.002	0.101	0.4936	89.31
490.3	66.0	−0.104	−0.024	−0.001	0.107	0.2768	86.19
600.4	110.1	0.106	0.014	−0.007	0.107	4.8351	111.45
724.5	124.1	0.115	−0.007	0.006	0.116	0.0928	139.76
798.5	74.1	−0.106	−0.002	0.010	0.107	1.4095	150.37
940.6	142.1	0.103	0.012	−0.003	0.103	3.0036	246.80
992.7	52.0	0.142	0.048	−0.007	0.150	3.3151	81.81
1064.7	72.0	0.119	−0.005	0.004	0.119	0.1081	232.10
1192.8	128.1	0.098	0.033	0.000	0.104	0.0475	69.41
1248.9	56.0	0.101	0.003	0.007	0.101	1.5651	163.81
1302.9	54.0	0.106	0.004	−0.004	0.106	1.8411	163.88
1384.9	82.1	0.113	0.029	0.003	0.116	2.1786	99.85
1443.0	58.0	0.102	−0.008	0.009	0.103	3.6649	198.15
1557.1	114.1	−0.104	−0.008	0.000	0.104	0.7582	184.31
1663.1	106.1	0.104	0.012	−0.005	0.104	0.0079	290.59
1723.2	60.0	0.163	0.057	0.002	0.173	3.4563	65.17
1771.2	48.0	0.111	0.005	0.005	0.111	0.7362	289.20
1863.3	92.1	0.101	0.022	−0.004	0.104	3.0167	366.10
2003.4	140.1	0.110	−0.013	0.001	0.111	0.9942	232.00
2083.4	80.1	0.314	0.097	0.010	0.329	2.8050	318.56
2145.5	62.0	0.106	0.001	0.005	0.106	0.3355	491.05

onboard propulsion system. (Note that ΔV_{min} denotes the magnitude of the smallest maneuver possible.) For all trials performed for the baseline cases, it was assumed $\Delta V_{min} = 0$; i.e., that a ΔV of any magnitude could be implemented with the modeled accuracy ($\sigma = 2.5\%$ of the planned maneuver magnitude, applied to $\dot{x}$, $\dot{y}$, $\dot{z}$ components). The smallest maneuver apparently executed by ISEE-3 while in its halo orbit was 0.5 m/s. It is noted that the ISEE-3 spacecraft, and most other planned libration point satellites, have the capability to perform much smaller maneuvers, i.e., about 0.01 m/s. The overall cost does generally increase as the value for ΔV_{min} is increased for group A, as expected. However, the cost for case B3 is slightly less than case B2, and the costs in group C decrease for the first three cases and then increase significantly for the final case (C5). At first view, this result is very surprising. However, upon reflection, a number of factors may contribute to such a result. For example, the input parameters have not been optimized, including the timing of each maneuver. The various ΔV_{min} values alter the timing of each maneuver, and this could unpredictably affect the total costs. The maneuver timing continually appears as a critical factor. The results for cases A2 and B2 are less than the corresponding costs for the baseline values in cases A1 and B1, which suggests that a delay in maneuver implementation may be helpful. Re-evaluating the impact of d_{min} and the assumption of maneuver implementation at $t = t_0$ may be a priority in any additional work. Note that the cost for case C1 is identical to that for case C2, since the magnitude of every ΔV (for each trial) was greater than 0.01 m/s for case C1. Thus, specifying $\Delta V_{min} = 0.01$ m/s had no effect on the simulations for case C2. The cause of the decreases in groups B and C is not immediately obvious, and additional studies may provide further insight into the effects of the parameter ΔV_{min}. It may also be appropriate to re-evaluate the error model. Although this analysis used the same error probabilities to determine maneuver errors, there may, in fact, be different appropriate σ values for various ΔV_{min}. For consideration of effects resulting from changes to other parameters, ΔV_{min} was returned to its baseline value of zero.

This study also investigated effects of varying the tracking interval, that is, varying the length of time between updates of tracking information. The stationkeeping algorithm incorporates the tracking interval by evaluating the "actual" spacecraft position and velocity relative to the nominal values only at specified time steps. (The baseline value was selected as 2 days.) If all constraints are met and it is determined that a maneuver is necessary, one is computed and implemented. The same evaluation occurs at the next tracking update time whether or not a maneuver was done. So "tracking interval" is essentially defined here as the evaluation interval. From the results shown in the table, the shortest interval—one day—appears to produce the lowest cost for groups A and B. However, an interval of 8 days results in the lowest cost for group C (case C8). This is surprising, since more frequent tracking intervals provide additional information and thus a reduction in maneuver cost was expected. The cause of this anomaly is not clear. There may be some self-correcting element of the system that has not been considered; a delay in maneuver implementation is again suggested. Also, more than 100 trials may impact relative results within this group. Additional studies are certainly required and could reveal unique characteristics contributing to the cost decrease seen for group C.

It is also of interest to consider the effects on the stationkeeping cost of an increase in injection or tracking errors. These results are displayed in detail in Table 2. The injection or tracking errors are increased by use of a "multiplication factor." For a given multiplication factor, the error value associated with each component of the state and obtained from the random-number generator using the baseline value for σ is multiplied by that factor. Thus a multiplication factor of one represents the baseline error quantities. Increasing the errors in this manner is equivalent to an increase in the standard deviation of the error source to the value produced by the product of the baseline σ and the multiplication factor. The section of the table labeled "injection errors" summarizes the resulting costs for simulations in which the injection error was multiplied by the indicated multiplication factor (with the tracking and maneuver errors fixed at their baseline values), and the "tracking errors" section provides resulting costs when the tracking errors were multiplied by the multiplication factor (with the injection and maneuver errors fixed at their baseline values). As the table indicates, five different multiplication factors (abbreviated "M.F." in the table) were considered: 20, 40, 60, 80, and 100. The baseline orbit injection error for one simulation, for example, was 12.55 km in position and 2.3 m/s in velocity, obtained randomly as just described. Thus, at a multiplication factor of 20, this error would increase to 251.0 km and 4.6 cm/s. Such an increase in injection error increases costs in each group, but for group C, in particular, the total average ΔV more than doubles. Note also that, in the table, some of the average maneuver cost figures are followed by a number in square brackets; "[4]," for example, in case C10. (This notation is in addition to the case numbers shown in parentheses.) In these cases, some of the trials resulted in trajectories that deviate farther than 50,000 km from the nominal path (resulting in termination of the simulation). The value in the brackets, then, is the number of diverging cases out of the 100 simulations that were performed. The average total maneuver costs listed do not include the diverging cases, and thus these averages were based on a number of simulations less than 100. (Case C10, then, is based on $100 - 4 = 96$ trials.) With injection errors returned to their baseline levels, results with increased tracking errors are also shown in the table. For the tracking errors obtained with σ at the baseline level, the largest tracking error incorporated during each simulation was typically on the order of 80 km in position and 1.6 cm/s in velocity. Thus the multiplication factor can cause a significant error in the position and velocity, in some cases on the order of 10,000 km and 2 m/s. Although the table displays multiplication factors as large as 100, the actual errors encountered by a spacecraft are likely to be much smaller. However, besides the impact of increasing errors on cost, displaying the effects of larger errors can provide information on the tolerance of the stationkeeping strategy to varying degrees of error magnitudes. The trajectory was successfully controlled for rather large errors, although some cases resulted in diverging trajectories. In particular, 73 of the 100 runs for case C15 diverged (corresponding to a multiplication factor of 40 for the tracking errors). Because of the frequently diverging trajectories, additional increases in the tracking errors for group C were not attempted. As the table indicates, the rise in cost is greater for increasing tracking errors compared to higher injection errors, for all three groups. Since injection is a one-time event and tracking involves recurring errors, this trend is logical.

Consistent with an investigation for increased injection and tracking errors, the possibility that maneuver errors might actually be higher than baseline values was also considered. A simple way to increase the errors is to multiply each component of the maneuver error, computed for the baseline σ, by some indicated factor for implementation of every ΔV. Some examples are shown in the last section of Table 2. It is typical to represent maneuver errors in terms of a percentage, i.e., the percentage error of the actual maneuver. The maneuver errors typically ranged from near-zero to about 7% at the baseline values of σ. At a multiplication factor of 6, the maneuver error percentages typically varied between near-zero and 40%. As the table indicates, maneuver errors above a certain level resulted in the spacecraft frequently drifting unacceptably far from the nominal path.

Although Table 2 displays the effects of varying some of the parameters associated with the stationkeeping procedure, a number of other parameters were fixed. For example, the number of future target times was fixed at two, and the time for implementation of a maneuver was fixed at time t_o. The effects of varying these and other parameters will be investigated in further studies.

Maneuver Histories

To demonstrate more completely stationkeeping maneuver histories that result from this strategy, a representative case from Table 2 has been arbitrarily selected for detailed presentation. The example is shown in Table 3, corresponding to the first trial for case A4. As indicated in Table 2, then, this trial assumed values $t_{\min} = 30$ days and $\Delta V_{\min} = 0.1$ m/s. The first column of Table 3, labeled "Time," is the time as measured from injection into the halo orbit. Then "Δt" is the length of time that has elapsed since the last maneuver. The following four columns then present the ΔV maneuver, in terms of it $\dot{x}$, $\dot{y}$, and $\dot{z}$ components and the resulting total magnitude. These four numbers represent the actual maneuver that was implemented and not the planned maneuver (i.e., the listed values for the maneuvers include the random errors). "Maneuver error %" indicates the error in execution of the planned maneuver. Finally, in the last column is the position deviation from the nominal path at the time of each maneuver. Note that the deviation is not the distance measured relative to the closest point on the nominal path, rather, it is the distance, defined previously as "d," from the nominal path at the time specified. For this trial, then, a total of 25 maneuvers were executed at a total cost of 3.014 m/s. Nearly all of the 25 maneuvers, however, barely exceed the minimum constraint value of 0.1 m/s (with the exception of the 24th maneuver—0.329 m/s, executed at 2083.4 days). It appears as though, in this case, minimum ΔV magnitude required for execution is usually the last constraint that must be satisfied before maneuver implementation. The shortest length of time between two successive maneuvers was 48.0 days and the longest 142.1 days. With $t_{\min} = 30$ days, the average length of time between the actual maneuvers was 85.8 days. It is interesting to note (and not unexpected) that for each maneuver, the magnitudes of the $\dot{y}$ and $\dot{z}$ components of each ΔV were much smaller than the magnitude of the $\dot{x}$ component. The maximum maneuver error was 4.84%, which occurred at 600.4 days (the 7th ΔV). Also, as indicated in the table, the maximum deviation from the nominal path when executing a maneuver was 491.1 km at 2145.5 days (the final maneuver). However, the maximum deviation over the entire simulation was 499.1 km.

Concluding Remarks

This work has involved the development of a stationkeeping strategy for spacecraft moving on halo or Lissajous orbits in the vicinity of the interior L_1 libration point of the Sun-Earth/Moon barycenter system. These preliminary results appear promising and suggest further development to improve the methodology. It is anticipated that the method will work for any system of primaries and for trajectories near the other libration points. Although maneuver costs are small using this technique, future efforts may involve strategies to further reduce ΔV requirements in more general applications.

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